the heater when the small flow visualization horizontal cylindrical tank was used.

3) The thermocline formed with a baffle placed adjacent to the line heat source, directly across from it, and with a baffle which was concentric with the small horizontal cylindrical tank having approximately one inch clearance between it and the wall.

4) The thermocline also formed when uniform wall heating was added to the line heat source heating in the small horizontal cylinder.

5) The thermocline formed with the container at several liquid fill levels.

In summary, these preliminary experiments have shown the likelihood of obtaining a thermocline at the level of a localized heat source both with and without container baffles. Although a parametric study of the liquid temperature-heat flux values was not made, indications were that this thermocline could be destroyed at liquid temperatures near the saturation temperature (without self pressurization) of the liquid with vigorous boiling heat flux values. In addition, the heating of a fraction of long cylindrical tank surface resulted in a similar sharp temperature gradient at the bottom edge of the heated portion of the tank, where self-pressurization resulted in a subcooled liquid below the surface of the liquid.

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Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure or vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

SERT II Hollow Cathode Multiple Restarts in Space

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Introduction

THE SERT II (Space Electric Rocket Test II) spacecraft was reactivated in May 1973 to demonstrate extended thruster cathode restarts in space. The original (1967) design goals of the SERT II thruster[1,2] did not call for the numerous restarts ($\sim$2000) required by many currently proposed electric thruster missions. Although the SERT II spacecraft was not programed to perform automatic thruster restarts, it was possible to manually command the thruster cathodes to light and then to turn them off. Constraints of ground base tracking stations schedules and spacecraft orbits limited restarting attempts to 30–40 per month.

In September 1973, a spacecraft ground-control room, necessary to continue restart testing of the SERT II thruster cathodes, was withdrawn from use by this program. The loss of this room prevented additional restarts from being attempted. This program, however, could not have been continued indefinitely due to a gradual loss of solar power. The initial sun-synchronous, polar orbit of the SERT II spacecraft had precessed such that a now oblique sun angle gave only marginal power to operate the cathodes. Wobbling of the spacecraft was already causing a critical time-varying available power in Sept. 1973 and submarginal power was predicted within a few months.

Thus, the test period available to this program was May through Aug. 1973, and 112 restarts of each cathode (two main and two neutralizer cathodes) were made during this test period. These tests showed no deterioration of cathode heaters, nor has any change been required in starting voltages and currents. In addition, restart after long periods (490 days) of space storage has been demonstrated.

This Note presents a summary of the cathode starting data obtained from the SERT II spacecraft during the above four month period and compares it with cathode starting data for the entire mission. For a more complete description of this program, test procedures, and cathode operational history, the reader is referredto the conference preprint[3] and earlier SERT II references.[4,5]

Flight Thruster Cathode Starting

Figure 1 chronologically shows the number of cathode restarts, storage time between, and total hours of operation. The cathodes were ground tested before launch in Dec. 1969, endurance tested in space during 1970, restarted numerous times through early 1972, and then stored (490 days) until May 1973. Restarts during the endurance phase were necessary because lunar eclipses of the sun caused temporary loss of solar cell power on board the spacecraft.

Figure 2 shows the time to start each cathode of flight thruster 1. (The data for thruster 2 are, in general, similar and are presented in Ref. 3.) All of the last 112 restarts are plotted along with representative starting times back to the original ground tests. The time between starts can be determined with the help of Fig. 1. The time to start is a parameter which indicates cathode starting reliability because each restart

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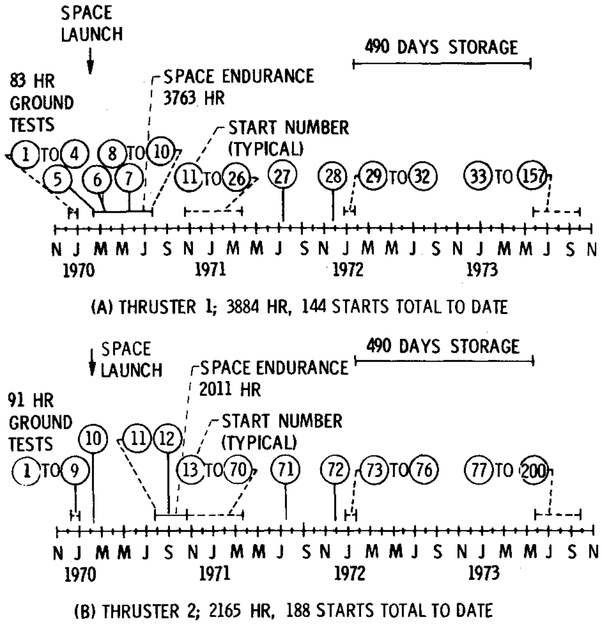

Fig. 1 Chronological representation of SERT II thruster preheats (starts), endurance running, and storage periods.

followed the same procedure and used the same starting voltage and heater currents.

No definite trend of start time developed over the last 112 start attempts on any of the four flight cathodes. The starting times ranged between 1 and 10 min (Fig. 2) and depended on component thermal time constants coupled with initial temperatures. The start times "early" in spacecraft life (start numbers 5–20 for thruster 1) were shorter corresponding to a higher initial temperature of the thrusters. This temperature was 78°–97°C in "early" life and was 22°–27°C for the last 112 attempts due to partial orbit eclipse. Both main cathodes, when preheated for 20 min or more, had a very short and consistent starting time of two minutes or less. These short starts can be seen in Fig. 2 for

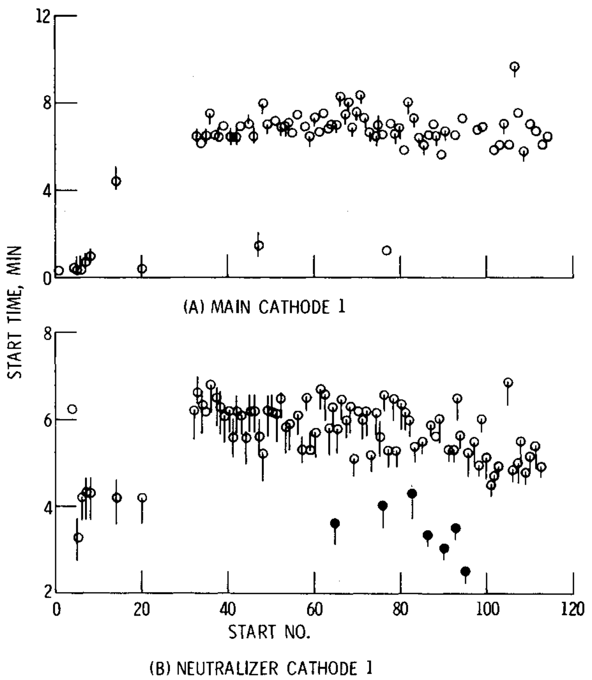

Fig. 2 Thruster 1; variation of cathode starting time. (Vertical bars represent telemetry data uncertainty time. Solid symbols, 2nd start during the same orbit.)

"early" life start attempts as well as those done recently, i.e., start numbers 47 and 77. The short starting time procedure was the strongest evidence that indicates that the main cathodes were continuing to light normally and consistently. This procedure holds true after 3884 hr (total operating time) with 144 starts and 2165 hr with 126 starts for thrusters 1 and 2, respectively. The error bars on Fig. 2 are the result of commutation of spacecraft data.

The space storage of SERT II hollow cathode affects their operation in no noticeable way. This is true for storage periods of 490 days as well as shorter periods of 50–150 days. (Storage periods can be seen in Fig. 1.) Consistent cathode starting times before/after storage, start numbers 32 and 76 for thruster 1, can be seen in Fig. 2 for the 490 day storage. Indeed, space storage was not expected to influence the starting ability of the SERT II-type hollow cathode, because the cathode derives its starting from the presence of a chemically active oxide compound. In the near perfect vacuum of space there are insignificant amounts of molecules, such as O_2 or H_2O, that can cause chemical deactivation of the oxide compound.

Ground Testing of Cathodes

Each flight thruster was qualified in ground testing before launch. Thruster 1 was started four times on the ground and thruster 2, nine times. Both thrusters were exposed to atmosphere four times between tests. The first time that a cathode is lighted, a longer period is needed for conversion of the carbonate in the insert and activation of the tip. Start number 1 for each thruster required the longest starting time for the neutralizer cathode. The initial start time of the main cathode was short because the 90-min thruster preheat cycle allows more than adequate time for cathode activation processes to occur.

Since no means were provided to measure cathode temperatures on the flight thrusters, separate ground tests were conducted in bell jars to measure cathode temperatures. These tests were done to provide estimates of cathode operation temperatures for the design of future thruster cathodes.[6] The cathode construction and heaters were identical with flight hardware and the cathode assemblies, mounted in a vacuum bell jar, closely simulated the thermal environment of an actual thruster. The neutralizer cathode tip temperatures at flight preheat currents reached 1200°–1300°C. The time constant of the tip heater, however, was 3 min, so that in 5–6 min of heating the tip, as for many of the last 112 neutralizer starts, the temperature was probably 100°C lower than the equilibrium values. After the cathode lights the tip heater power was reduced, resulting in a cathode tip temperature of approximately 1100°C. The main cathode temperature during preheat was about 50°C hotter than the neutralizer because of the heat applied to a nearby isolator heater.

Discussion of Cathode Life

The results of this Note indicate that hollow cathodes can be designed and operated to meet space thruster mission requirements of thousands of hours operation and hundreds of restarts. Future cathode design operation should be equivalent to or lower than the SERT II conditions, i.e., preheat temperatures of 1150°–1300°C and operating temperatures equal or less than this range. The potential life-limiting processes in hollow cathodes of the SERT II design are 1) erosion of the tip, 2) exhaustion of the insert active material, and 3) heater failure.

Erosion of the tip has been reduced to a negligible level as evidenced by observations following ground life tests of the SERT II thrusters[4,7] and by technology life tests of 30-cm and 5-cm diam thruster cathodes.[8,9]

The cathode inserts were removed and analyzed for BaO and SrO content following the conclusion of two SERT II thruster ground life tests.[4] In addition, a new cathode insert was analyzed at the same time by the identical technique. The results of the analysis indicated that the cathode inserts, operated 5412 and 6787 hr, respectively, in the life tests still contained a major portion of the BaO and SrO found in a new insert.

Exhaustion of insert active material or heater failure problems have been nonexistent in the SERT II flight thrusters or ground life tests to date. This should continue to be so in future thrusters if correct operating temperatures and heater fabrication procedures are followed.

The reliability of the cathode heaters can be inferred from the consistency of the currents, voltages, and resistances (ratio of heating voltage-to-current). Reference 3 presents these values for all of the flight heaters, and the major variation ($\pm 3\%$) of the values were due to quantizing of the spacecraft data. Within this variation, the authors interpret the heater values to indicate no degradation of any SERT II heater.

Summary

SERT II spacecraft data taken during the summer of 1973 indicated no starting degradation in any of the four hollow cathodes on board the spacecraft. Total hours of cathode operation for flight thrusters 1 and 2 were 3884 hr and 2165 hr with 144 and 188 restarts, respectively. Restarting was also accomplished after space storage periods up to 490 days. These data indicate that with proper design and operation a hollow cathode thruster will operate and restart for the long times and many restarts required by future missions.

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Similarity Laws for Missiles of Minimum Ballistic Factor

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Introduction

THE problem of determining the geometry of a slender axisymmetric body having a minimum ballistic factor has attracted considerable attention during the recent past.[1–5] Tawakley and Jain[6,7] have used the method of extremizing the product of powers of integrals to find the shapes of slender axisymmetric bodies having a minimum ballistic factor in Newtonian hypersonic flow when any two of the three quantities of length, diameter, and surface area are known in advance. In the case of minimizing the drag, Miele[8] has shown that a) a similarity law exists which enables determination of the optimum longitudinal contour of a body of arbitrary transversal contour from the known optimal longitudinal contour of a reference body, and b) a similarity law exists which enables determination of the optimum transversal contour of a body of arbitrary longitudinal contour from the known optimum transversal contour of a reference body. It is shown here that these similarity laws also exist for determining the minimum ballistic factor body shapes. The main assumptions are that the distribution of pressure coefficient is Newtonian, the skin-friction coefficient is constant, the body is slender in the longitudinal sense, and the body is homothetic (i.e., each cross section is geometrically similar to the base cross section and has the same orientation).

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Aerodynamic and Geometric Quantities

Let the shape of the body be represented in cylindrical coordinates (x, r, θ) by the equation

$$f(x, r, \theta) = 0 \tag{1}$$

where x is the direction of the freestream, r is the distance of any point from the x axis and θ gives the angular position of this point with respect to some plane, and $\theta = 0$. Then, for a slender body in the longitudinal sense, the drag, the surface area, and the volume are given by[9]

$$\frac{D}{q} = \int_0^l \int_0^{2\pi} \frac{r}{f_r}\left[-\frac{2f_x^3}{f_r^2+(f_\theta/r)^2} + C_f\,[f_r^2+(f_\theta/r)^2]^{1/2} dx\,d\theta\right. \tag{2}$$

$$S = \int_0^l \int_0^{2\pi} \left(\frac{r}{f_r}\right)[f_r^2+(f_\theta/r)^2]^{1/2} dx\,d\theta \tag{3}$$

$$V = \frac{1}{2}\int_0^l \int_0^{2\pi} r^2 dx\,d\theta \tag{4}$$

where q is the freestream dynamic pressure, l is the length of the body, and C_f is the constant skin-friction coefficient.

Since the body is supposed to be homothetic, Equation (1) is of the form

$$r = A(x)B(\theta) \tag{5}$$

where A and B denote arbitrary specified functions of x and θ, respectively. $A(x)$ describes the longitudinal contour and is such that $A(l) = 1$, whereas $B(\theta)$ describes the cross-sectional contour and is such that $B(0) = 1$. By making use of Eq. (5), the drag, the surface area, and the volume can be written as

$$\frac{D}{q} = 2\int_0^l A\dot{A}^3 dx \int_0^{2\pi} \frac{B^6}{B^2+\dot{B}^2} d\theta + C_f \int_0^l A dx \int_0^{2\pi} (B^2+\dot{B}^2)^{1/2} d\theta \tag{6}$$

$$S = \int_0^l A dx \int_0^{2\pi} (B^2+\dot{B}^2)^{1/2} d\theta \tag{7}$$

$$V = \frac{1}{2}\int_0^l A^2 dx \int_0^{2\pi} B^2 d\theta \tag{8}$$

where the dot represents the derivative with respect to the functional variable.

The ballistic coefficient of a body is proportional[2] to the ratio D/qV which may be represented by C (defined as "quality coefficient"). Therefore

$$C = \frac{D}{qV} = \frac{4\int_0^l A\dot{A}^3 dx \int_0^{2\pi} \frac{B^6}{B^2+\dot{B}^2} d\theta + 2C_f \int_0^l A dx \int_0^{2\pi} (B^2+\dot{B}^2)^{1/2} d\theta}{\int_0^l A^2 dx \int_0^{2\pi} B^2 d\theta} \tag{9}$$

The possible constraints can be on the drag, the length, the diameter or thickness, the surface area, and the volume, but